Theoretical Analysis of Acoustic Signal Generation in Materials Irradiated with Microwave Energy

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Abstract—Stress gradients generated by thermal expansion, electrostriction, and radiation pressure are sources of elastic waves in microwave irradiated materials. A theoretical analysis taking into account induced volume and surface forces due to these interaction mechanisms is presented. Complete solutions of the dynamical equations for the one-dimensional special case are given for different boundary conditions. The closed-form solutions were found to consist of both a stationary part, whose effect is important only in the immediate region of the incident electromagnetic wave, and a traveling part which propagates through the elastic material. Expressions for the Fourier transforms of these solutions are also given. To quantify these results, pressure and displacement waveforms in microwave irradiated physiological saline were computed. Thermal expansion was considerably more effective than either electrostriction or radiation pressure in converting electromagnetic energy to acoustic energy.

I. Introduction

THE RAPID ABSORPTION of electromagnetic energy in irradiated elastic media can result in easily detectable acoustic signals. Stress waves generated in absorbing media due to interaction with laser radiation have been studied by a number of investigators [1]–[5], [9]. In addition, acoustic transients can be generated in irradiated materials by pulsed electron beams [8]–[10], or by pulses of microwave radiation [9], [11]–[13].

Human subjects hear a distinct "click" when the head is irradiated with a high-energy microwave pulse [13]–[15]. Microwave induced acoustic effects in other mammalian auditory systems have been reported [13], [16]. It is likely that the "hearing" effect and the generation of acoustic signals in irradiated materials are closely related phenomena.

Several physical mechanisms have been suggested for the energy transfer between an electromagnetic wave and the induced elastic wave in the interacting medium [1]–[13]. In this paper, a theoretical analysis is presented in which the following likely candidates for the causative mechanism are considered: the volume forces due to thermal stress and electrostriction, and the surface force due to radiation pressure.

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II. Theoretical Analysis

To facilitate a quantitative analysis, the problem will be considered within the framework of the following simplifying assumptions. The irradiated media are assumed to be electrically and elastically homogeneous and isotropic. Only a small fraction of the incident radiation ultimately appears as acoustic energy. Acoustic attenuation will be neglected and uncoupled thermoelastic theory will be employed, i.e., the temperature distribution will be obtained by solution of the heat equation independent of the mechanical state of the medium. Finally, all strain components are assumed to be small enough so that their squares and products are negligible.

In what follows, let σ_{ij} and e_{ij} denote the components of the stress and strain tensors, respectively, for an electrically and elastically homogeneous and isotropic material. Let $\theta(x,t)$ denote the increment of temperature above some reference temperature for which the material is in a state of zero stress and strain. The strain components are defined in terms of the displacement vector $\boldsymbol{u}$ having components u_1, u_2, and u_3 according to [17]:

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{1}$$

Thermal Expansion

The components of the stress tensor relating stress to strain and excess temperature are [17], [18]

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e_{kk}\delta_{ij} - \beta\theta(x,t)\delta_{ij} \tag{2}$$

where δ_{ij} is the Kronecker delta, λ and μ are the usual Lamé constants, and

$$e_{kk} = e_{11} + e_{22} + e_{33}. \tag{3}$$

If α is the coefficient of linear thermal expansion and B is the bulk modulus of the material, then [17]

$$\beta = (3\lambda + 2\mu)\,\alpha = 3B\alpha. \tag{4}$$

Electrostriction

Let E_i denote the components of the electric field intensity $\boldsymbol{E}$ in the elastic dielectric. The electrostrictive effect is the deformation of a dielectric medium whose accompanying strains are proportional to the even powers of the electric field intensity. The basic field equations governing electrostriction have been derived from thermodynamic arguments [19], [20].

The most general linear relationship that can be constructed between stress, strain, and the quadratic powers of E_i is [21]

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e_{kk}\delta_{ij} + aE_iE_j + bE_kE_k\delta_{ij} \tag{5}$$

where a and b are descriptive of the electrical properties of the dielectric and are expressed below in terms of measurable quantities, and

$$E_kE_k = E_1^2 + E_2^2 + E_3^2. \tag{6}$$

Under isothermal conditions only the strains influence the anisotropy of the deformed dielectric. Because the strains are assumed to be linear, we may write the components of the dielectric tensor [20], [21]:

$$\varepsilon_{ij} = \varepsilon\delta_{ij} + \varepsilon_1 e_{ij} + \varepsilon_2 e_{kk}\delta_{ij} \tag{7}$$

where ε is the permittivity of the undeformed media. The scalar constants ε_1 and ε_2 are characteristics of the electrostrictive property and, except in special cases, must be determined experimentally. Constants a and b are related to ε_1 and ε_2 by [20], [21]

$$a = 2\varepsilon - \varepsilon_1 \qquad b = -(\varepsilon_1 + \varepsilon_2). \tag{8}$$

Dynamical Equations

The differential equations of motion of an elastic body are [17], [18]

$$\begin{aligned}
\frac{\partial\sigma_{11}}{\partial x_1} + \frac{\partial\sigma_{12}}{\partial x_2} + \frac{\partial\sigma_{13}}{\partial x_3} + F_1 &= \rho\frac{\partial^2 u_1}{\partial t^2}\\
\frac{\partial\sigma_{12}}{\partial x_1} + \frac{\partial\sigma_{22}}{\partial x_2} + \frac{\partial\sigma_{23}}{\partial x_3} + F_2 &= \rho\frac{\partial^2 u_2}{\partial t^2}\\
\frac{\partial\sigma_{13}}{\partial x_1} + \frac{\partial\sigma_{23}}{\partial x_2} + \frac{\partial\sigma_{33}}{\partial x_3} + F_3 &= \rho\frac{\partial^2 u_3}{\partial t^2}
\end{aligned} \tag{9}$$

where ρ is the density of the medium and F_i are components of the body force $\boldsymbol{F}$.

The dynamical equations of thermoelasticity can be derived by substituting (2) into (9). Assuming body force $\boldsymbol{F}$ is zero and all partial derivatives are continuous, we obtain

$$\begin{aligned}
\mu\nabla^2 u_1 + (\lambda + \mu)\frac{\partial e_{kk}}{\partial x_1} - \beta\frac{\partial}{\partial x_1}\theta &= \rho\frac{\partial^2 u_1}{\partial t^2}\\
\mu\nabla^2 u_2 + (\lambda + \mu)\frac{\partial e_{kk}}{\partial x_2} - \beta\frac{\partial}{\partial x_2}\theta &= \rho\frac{\partial^2 u_2}{\partial t^2}\\
\mu\nabla^2 u_3 + (\lambda + \mu)\frac{\partial e_{kk}}{\partial x_3} - \beta\frac{\partial}{\partial x_3}\theta &= \rho\frac{\partial^2 u_3}{\partial t^2}.
\end{aligned} \tag{10}$$

A more compact vector form of (10) is

$$\mu\nabla^2\boldsymbol{u} + (\lambda + \mu)\nabla(\nabla\cdot\boldsymbol{u}) - \beta\nabla\theta = \rho\frac{\partial^2\boldsymbol{u}}{\partial t^2} \tag{11}$$

where

$$\nabla\cdot\boldsymbol{u} = e_{kk}. \tag{12}$$

The dynamical equations of electrostriction can be obtained by substituting (5) into (9). Again, assuming body force $\boldsymbol{F}$ is zero and all partial derivatives are continuous, we obtain

$$\begin{aligned}
&\mu\nabla^2 u_1 + (\lambda + \mu)\frac{\partial}{\partial x_1}e_{kk} + a\left[\frac{\partial}{\partial x_1}E_1^2 + \frac{\partial}{\partial x_2}E_1E_2 + \frac{\partial}{\partial x_3}E_1E_3\right] + b\frac{\partial}{\partial x_1}E_kE_k = \rho\frac{\partial^2 u_1}{\partial t^2}\\
&\mu\nabla^2 u_2 + (\lambda + \mu)\frac{\partial}{\partial x_2}e_{kk} + a\left[\frac{\partial}{\partial x_1}E_1E_2 + \frac{\partial}{\partial x_2}E_2^2 + \frac{\partial}{\partial x_3}E_2E_3\right] + b\frac{\partial}{\partial x_2}E_kE_k = \rho\frac{\partial^2 u_2}{\partial t^2}\\
&\mu\nabla^2 u_3 + (\lambda + \mu)\frac{\partial}{\partial x_3}e_{kk} + a\left[\frac{\partial}{\partial x_1}E_1E_3 + \frac{\partial}{x_2}E_2E_3 + \frac{\partial}{\partial x_3}E_3^2\right] + b\frac{\partial}{\partial x_3}E_kE_k = \rho\frac{\partial^2 u_3}{\partial t^2}.
\end{aligned} \tag{13}$$

On Determining $\theta(x,t)$

In a region where energy is being dissipated as heat, $\theta(x,t)$ can be determined by solving, with appropriate boundary and initial conditions, the heat conduction equation [22], [23]

$$\nabla^2\theta(x,t) + \frac{h(x,t)}{K} = \frac{1}{\kappa}\frac{\partial\theta(x,t)}{\partial t} \tag{14}$$

where κ is the thermal diffusivity, K is the thermal conductivity, and $h(x,t)$ is the energy dissipated as heat per unit volume per unit time. In a material exposed to a time-periodic electromagnetic wave, the mean heat flux is [19]

$$h(x,t) = -\mathrm{Re}\,\nabla\cdot\boldsymbol{P} \tag{15}$$

where P is the complex Poynting vector

$$\boldsymbol{P} = \tfrac{1}{2}\boldsymbol{E}\times\boldsymbol{H}^* \tag{16}$$

$\boldsymbol{H}$ is the magnetic field intensity, and the asterisk denotes the conjugate. It should be noted that the expression for $h(x,t)$ as written in (15) suppresses the harmonic part of heat flux in a material exposed to a time-periodic electromagnetic wave. The harmonic term results in a temperature component varying at the frequency 2ω, where ω is the angular frequency of the incident electromagnetic wave [9]; in what follows we consider only the mean value of $h(x,t)$ as expressed in (15).

One-Dimensional Problem

Assume a plane electromagnetic wave is incident normally on a semi-infinite body which is elastically and dielectrically homogeneous and isotropic. The boundary surface of the irradiated material is formed by the plane $x_1 = 0$. Thus the Poynting vector is directed along the x_1 axis.

In this one-dimensional problem, derivatives with respect to x_2 and x_3 vanish; hence

$$e_{22} = e_{33} = 0. \tag{17}$$

For simplicity assume the plane wave is linearly polarized along the x_2 axis. Then,

$$E_1 = E_3 = 0 \quad \text{and} \quad H_1 = H_2 = 0. \tag{18}$$

Thus, from (10) and (13), we obtain the following nonhomogeneous one-dimensional wave equations for the thermoelastic and electrostrictive cases, respectively:

$$\rho \frac{\partial^2 u_1}{\partial t^2} - \rho v^2 \frac{\partial^2 u_1}{\partial x_1^2} = -\beta \frac{\partial}{\partial x_1} \theta \tag{19}$$

and

$$\rho \frac{\partial^2 u_1}{\partial t^2} - \rho v^2 \frac{\partial^2 u_1}{\partial x_1^2} = (a + b) \frac{\partial}{\partial x_1} E_2^2 \tag{20}$$

where v, the velocity of elastic wave propagation, is related to the Lamé constants by

$$\rho v^2 = (\lambda + 2\mu). \tag{21}$$

In a fluid it can be shown (see [19, p. 145]) that

$$a = 0 \quad \text{and} \quad b = -\tfrac{1}{2}\rho \frac{\partial \varepsilon}{\partial \rho}. \tag{22}$$

Making use of the Clausius–Mossotti law [19],

$$b = -\frac{\varepsilon_0}{6}(\varepsilon_r - 1)(\varepsilon_r + 2) \tag{23}$$

where ε_r is the relative dielectric constant and ε_0 is the permittivity of free space. Therefore, for a fluid dielectric, (20) can be written as

$$\frac{\partial^2 u_1}{\partial t^2} - v^2 \frac{\partial^2 u_1}{\partial x_1^2} = \frac{\varepsilon_0}{6\rho}(\varepsilon_r - 1)(\varepsilon_r + 2) \frac{\partial}{\partial x_1} E_2^2. \tag{24}$$

The bulk modulus B in a fluid can be expressed as

$$B = \rho v^2 \tag{25}$$

so that, using (25) and (4), (19) can be written as

$$\frac{\partial^2 u_1}{\partial t^2} - v^2 \frac{\partial^2 u_1}{\partial x_1^2} = -3v^2 \alpha \frac{\partial}{\partial x_1} \theta. \tag{26}$$

A time-periodic electromagnetic wave is often expressed in terms of the mean intensity of the energy flow $\boldsymbol{I}(x,t)$ where

$$\boldsymbol{I}(x,t) = \text{Re}\, \boldsymbol{P} = \tfrac{1}{2} \text{Re}\, (\boldsymbol{E} \times \boldsymbol{H}^*). \tag{27}$$

For the one-dimensional problem, the transmitted mean intensity in the elastic dielectric is

$$I_T(x_1,t) = \frac{1}{2} \frac{E_2^2}{\text{Re}\, \eta} \tag{28}$$

where η is the intrinsic impedance of the material.

The excess temperature, a function only of x_1 and time, can be computed by solving the one-dimensional heat conduction equation

$$\frac{\partial^2}{\partial x_1^2} \theta(x_1,t) + \frac{h(x_1,t)}{K} = \frac{1}{\kappa} \frac{\partial \theta(x_1,t)}{\partial t} \tag{29}$$

using the appropriate boundary and initial conditions. From (15) and (27),

$$h(x_1,t) = -\text{Re}\, \nabla \cdot \boldsymbol{P} = -\frac{\partial}{\partial x_1} I_T(x_1,t). \tag{30}$$

Rectangular Microwave Pulses

For rectangular incident pulses the transmitted intensity has the form

$$I_T(x_1,t) = I_0 e^{-2\gamma x_1}, \quad 0 < t < T$$
$$= 0, \quad \text{elsewhere} \tag{31}$$

where I_0 is the power density at $x_1 = 0$, γ is the reciprocal of the skin depth at a given frequency, and T is the pulsewidth.

The mean heat flux $h(x_1,t)$ can be computed from (30) and (31):

$$h(x_1,t) = 2\gamma I_0 e^{-2\gamma x_1}, \quad 0 < t < T$$
$$= 0, \quad \text{elsewhere.} \tag{32}$$

During the duration of the rectangular pulse ($0 < t < T$), the excess temperature $\theta(x_1,t)$, the solution of (29) with (32) as the forcing function, is [22], [24]

$$\theta(x_1,t) = \frac{2I_0}{K} \phi \, \text{ierfc} \left[\frac{x_1}{2\phi}\right] - \frac{I_0}{2\gamma K} e^{-2\gamma x_1} + \frac{I_0}{4K\gamma} e^{(4\gamma^2\phi^2 - 2\gamma x_1)} \cdot \text{erfc}\left[2\gamma\phi - \frac{x_1}{2\phi}\right] + \frac{I_0}{4\gamma K} e^{(4\gamma^2\phi^2 + 2\gamma x_1)} \cdot \text{erfc}\left[2\gamma\phi + \frac{x_1}{2\phi}\right] \tag{33}$$

where $\phi = \sqrt{t\kappa}$, erf is the error function, erfc $= 1 -$ erf, and

$$\text{ierfc}\,(z) = \int_z^\infty \text{erfc}\,(t)\, dt.$$

For short microwave pulses where heat conduction can be ignored,

$$\theta(x_1,t) = \frac{1}{\rho S} \int_0^t h(x_1,t)\, dt = \frac{2\gamma t I_0 e^{-2\gamma x_1}}{\rho S}, \quad 0 < t < T \tag{34}$$

where S is the specific heat of the material. It can be shown that (33) reduces to (34) when $x_1/2\phi \gg 1$ and $4\gamma^2\phi^2 \ll 1$ (see Appendix).

In nonmetallic materials, the elastic transients propagate away from the heated region in microseconds while it takes milliseconds for the temperature to decay. Therefore for $t > T$ we assume

$$\theta(x_1,t) \simeq \frac{2\gamma T I_0 e^{-2\gamma x_1}}{\rho S}, \quad t > T. \tag{35}$$

Substituting (34) and (35) into (26) we obtain

$$\frac{\partial^2 u_1}{\partial t^2} - v^2 \frac{\partial^2 u_1}{\partial x_1^2} = K_1 t e^{-2\gamma x_1}, \qquad 0 < t < T$$
$$= K_1 T e^{-2\gamma x_1}, \qquad t > T$$
$$= 0, \qquad t < 0 \tag{36}$$

where

$$K_1 = \frac{12\gamma^2 v^2 \alpha I_0}{\rho S}. \tag{37}$$

For the electrostrictive case assuming a fluid material,

$$\frac{\partial^2 u_1}{\partial t^2} - v^2 \frac{\partial^2 u_1}{\partial x_1^2} = K_2 e^{-2\gamma x_1}, \qquad 0 < t < T \tag{38}$$
$$= 0, \qquad \text{elsewhere}$$

where

$$K_2 = \frac{2\gamma \text{ Re } \eta \varepsilon_0 (\varepsilon_r - 1)(\varepsilon_r + 2) I_0}{3\rho}. \tag{39}$$

For rectangular microwave pulses, solutions to (36) and (38) will now be given subject to appropriate boundary conditions.

III. Solutions

In the solutions to (36) and (38) which follow, expressions will be given for both displacement $u_1(x_1,t)$ and pressure where the pressure $p(x_1,t)$ is simply the negative of the normal stress $\sigma(x_1,t)$. Solutions will include both the stationary and traveling wave terms for both free- and contrained-surface boundary conditions.

Boundary and Initial Conditions

For the thermal-expansion mechanism, the relation governing acoustic pressure and displacement may be found from the one-dimensional form of (2):

$$p(x_1,t) = -(\lambda + 2\mu) \frac{\partial u_1(x_1,t)}{\partial x_1} + \beta \theta(x_1,t). \tag{40}$$

Combining (40) with the results of (4), (21), and (25), the following relationship between $p(x_1,t)$, $u_1(x_1,t)$, and $\theta(x_1,t)$ may be found:

$$p(x_1,t) = -\rho v^2 \frac{\partial u_1(x_1,t)}{\partial x_1} + 3\rho v^2 \alpha \theta(x_1,t). \tag{41}$$

Finally, substituting the results of (34) and (35) into (41), the pressure–displacement relationship for the thermal-expansion mechanism may be found:

$$p(x_1,t) = -\rho v^2 \frac{\partial u_1(x_1,t)}{\partial x_1} + \frac{6\alpha\gamma v^2 I_0}{S} e^{-2\gamma x_1} \cdot (tU(t) - (t-T)U(t-T)) \tag{42}$$

where $U(t)$ is the unit step function.

In a similar manner, the relation governing acoustic pressure and displacement may be found for the electrostriction mechanism. The one-dimensional form of (5) is

$$p(x_1,t) = -(\lambda + 2\mu) \frac{\partial u_1(x_1,t)}{\partial x_1} - (a+b)E_2^2. \tag{43}$$

Substituting (21)–(23) and (28) into (43), we obtain

$$p(x_1,t) = -\rho v^2 \frac{\partial u_1(x_1,t)}{\partial x_1} + \frac{I_T(x_1,t)\varepsilon_0(\varepsilon_r - 1)(\varepsilon_r + 2) \text{ Re } [\eta]}{3}. \tag{44}$$

Finally, substituting the results of (31) into (41), the pressure–displacement relationship for the electrostriction mechanism may be found:

$$p(x_1,t) = -\rho v^2 \frac{\partial u_1(x_1,t)}{\partial x_1} + \frac{I_0 \varepsilon_0 (\varepsilon_r - 1)(\varepsilon_r + 2) \text{ Re } [\eta]}{3} \cdot e^{-2\gamma x_1}[U(t) - U(t-T)]. \tag{45}$$

For a constrained surface, the boundary conditions are

$$u_1(0,t) = 0 \qquad \lim_{x_1 \to \infty} |u_1(x_1,t)| < \infty \tag{46}$$

which hold for both mechanisms.

For a free surface, an additional mechanism for generating acoustic waves must be incorporated into the boundary conditions. A surface radiation pressure is now present on the materials and forms a part of the free-surface boundary conditions. For both bulk mechanisms, the general boundary conditions are

$$p(0,t) = \begin{cases} \dfrac{2I_r(0,t)}{c}, & 0 < t < T \\ 0, & t > T \end{cases} \tag{47}$$

$$\lim_{x_1 \to \infty} |u_1(x_1,t)| < \infty \tag{48}$$

where $I_r(0,t)$ is the intensity of the reflected wave at the surface and c is the speed of light. For thermal expansion, (42) combined with (47) becomes

$$\left. \frac{\partial u_1(x_1,t)}{\partial x_1} \right|_{x_1=0} = \frac{6\alpha\gamma I_0}{\rho s}[tU(t) - (t-T)U(t-T)] - \frac{2I_r(0,t)}{\rho v^2 c}[U(t) - U(t-T)]. \tag{49}$$

For electrostriction, (45) combined with (47) becomes

$$\left. \frac{\partial u_1(x_1,t)}{\partial x_1} \right|_{x_1=0} = \left[\frac{I_0 \text{ Re } [\eta] \varepsilon_0 (\varepsilon_r - 1)(\varepsilon_r + 2)}{3\rho v^2} - \frac{2I_r(0,t)}{\rho v^2 c} \right] \cdot [U(t) - U(t-T)]. \tag{50}$$

In all cases the initial conditions are

$$\left. \frac{\partial u_1(x_1,t)}{\partial t} \right|_{t=0} = \left. u_1(x_1,t) \right|_{t=0} = 0. \tag{51}$$

Solution Methods

In addition to finding the solution to (36) and (38) in terms of the displacement $u_1(x_1,t)$, the complete analysis of the electromagnetic field induced acoustic waves should include finding the pressure $p(x_1,t)$ as well as the magnitude of the Fourier transforms of $u_1(x_1,t)$ and $p(x_1,t)$. In the following, equations (36) and (38) were first solved for the displacement $u_1(x_1,t)$ and the corresponding pressure function $p(x_1,t)$ was then found using equation (42) or (45). After $u_1(x_1,t)$ and $p(x_1,t)$ have been determined, the Fourier transforms of these quantities can then be taken.

Equations (36) and (38) may be solved using several different techniques. One technique, used by Hu [6] in solving a similar problem, involves first removing the time dependency via a Laplace transformation and then finding the time-independent Green function under the given boundary conditions. The time-independent solutions to (36) and (38) are then found by the use of superposition of the Green function with the spacial source distribution. Finally, applying an inverse Laplace transform to the time-independent solution restores the time dependency to the solution.

Considering the geometry of the problem, a somewhat simpler approach is to utilize the technique used by Gournay [4]. First the equation together with the appropriate boundary and initial conditions is Laplace transformed to remove the time dependency. The resulting equation is now a second-order ordinary differential equation in space, which is easily solved by finding the homogeneous and particular solution using the standard techniques of ordinary differential equations. The boundary and initial conditions are then applied to the time-independent solution. The inverse Laplace transform is then taken to yield the complete solutions to (36) and (38). An additional benefit of this technique is the ease in obtaining the Fourier transform of $u_1(x_1,t)$ and $p(x_1,t)$. Providing the region of convergence of the Laplace transform of $u_1(x_1,t)$ or $p(x_1,t)$ contains the $j\omega$ axis in its interior, a simple transformation of s to $j\omega$ converts the Laplace transform to the Fourier transform [25]. This technique has been used to find the solutions given below.

The displacements and pressures, due to each of the various electromagnetic-to-acoustic wave mechanisms discussed above, will be given in three parts below, for both the free-surface and constrained-surface boundary conditions. In the first part, the displacement and pressure due to the bulk thermal-expansion mechanism will be found for both boundary conditions. Next, the displacements and pressure due to the electrostriction mechanism for the two boundary conditions will be found. Finally, the magnitude functions of the Fourier transforms of $u_1(x_1,t)$ and $p(x_1,t)$ will be determined for all the electromagnetic-to-acoustic energy mechanisms considered above, for both free- and constrained-surface boundary conditions.

Thermal Expansion

Constrained Surface: If we assume the surface of the material to be constrained, (36) must be solved subject to the aforementioned boundary and initial conditions. Subject to these conditions the solution to equation (36) may be written in two parts: a stationary part denoted by $u_{1_{ST}}$ and a traveling part denoted by $u_{1_{TR}}$. Therefore, the displacement due to thermal expansion becomes

$$u_1(x_1,t) = u_{1_{ST}}(x_1,t) + u_{1_{TR}}(x_1,t) \tag{52}$$

where

$$u_{1_{ST}}(x_1,t) = \frac{3\alpha I_0}{\rho S}\left[-te^{-2\gamma x_1}U(t) + (t-T)e^{-2\gamma x_1}U(t-T)\right] \tag{53}$$

and

$$\begin{aligned} u_{1_{TR}}(x_1,t) = \frac{3\alpha I_0}{\rho s}\Bigg\{ & \left[\frac{e^{-2\gamma x_1}}{2\gamma v}\sinh 2\gamma vt\right]U(t) \\ & + \left[\left(t-\frac{x_1}{v}\right) - \frac{1}{2\gamma v}\sinh 2\gamma v\left(t-\frac{x_1}{v}\right)\right] \\ & \cdot U\left(t-\frac{x_1}{v}\right) \\ & - \left[\frac{e^{-2\gamma x_1}}{2\gamma v}\sinh 2\gamma v(t-T)\right]U(t-T) \\ & + \left[\left(-t+T+\frac{x_1}{v}\right)\right. \\ & \left. + \frac{1}{2\gamma v}\sinh 2\gamma v\left(t-T-\frac{x_1}{v}\right)\right] \\ & \cdot U\left(t-T-\frac{x_1}{v}\right)\Bigg\}. \end{aligned} \tag{54}$$

The pressure $p(x_1,t)$ due to thermal expansion in a fluid is given by (42). A general expression for the pressure wave with a thermal-stress forcing function is

$$\begin{aligned} p(x_1,t) = \frac{3\alpha v I_0}{S}\Bigg\{ & [e^{-2\gamma x_1}\sinh 2\gamma vt]U(t) \\ & + \left[1-\cosh 2\gamma v\left(t-\frac{x_1}{v}\right)\right] \\ & \cdot U\left(t-\frac{x_1}{v}\right) - [e^{-2\gamma x_1}\sinh 2\gamma v(t-T)]U(t-T) \\ & + \left[\cosh 2\gamma v\left(t-T-\frac{x_1}{v}\right)-1\right]U\left(t-T-\frac{x_1}{v}\right)\Bigg\}. \end{aligned} \tag{55}$$

Free Surface: For a free surface, considering only thermal-expansion effects, the boundary condition on the materials is (49). Solving (36) under these conditions for the displacement, $u_1(x_1,t)$ is given by (52), where $u_{1_{ST}}(x_1,t)$ is given by

(53) and $u_{1_{TR}}(x_1,t)$ is given by

$$u_{1_{TR}}(x_1,t) = \frac{3\alpha I_0 e^{-2\gamma x_1}}{2v\gamma\rho S}\{[\sinh 2\gamma vt]U(t) - [\sinh 2\gamma v(t-T)]U(t-T)\} + \left[\frac{2I_r(0,t)}{\rho vc}\left(t-\frac{x_1}{v}\right) + \frac{3\alpha I_0}{2v\gamma\rho S} \cdot \left(1-\cosh 2\gamma v\left(t-\frac{x_1}{v}\right)\right)\right]U\left(t-\frac{x_1}{v}\right) - \left[\frac{2I_r(0,t)}{\rho vc}\left(t-T-\frac{x_1}{v}\right) + \frac{3\alpha I_0}{2v\gamma\rho S} \cdot \left(1-\cosh 2\gamma v\left(t-T-\frac{x_1}{v}\right)\right)\right] \cdot U\left(t-T-\frac{x_1}{v}\right). \quad (56)$$

Using (42) the general expression for the pressure wave for a free-surface boundary condition is

$$p(x_1,t) = \frac{3\alpha v I_0 e^{-2\gamma x_1}}{S}\{[\sinh 2\gamma vt]U(t) - [\sinh 2\gamma v(t-T)]U(t-T)\} + \left[\frac{2I_r(0,t)}{c} - \frac{3\alpha v I_0}{S}\sinh 2\gamma v\left(t-\frac{x_1}{v}\right)\right] \cdot U\left(t-\frac{x_1}{v}\right) - \left[\frac{2I_r(0,t)}{c} - \frac{3\alpha v I_0}{S}\sinh 2\gamma v\left(t-T-\frac{x_1}{v}\right)\right] \cdot U\left(t-T-\frac{x_1}{v}\right). \quad (57)$$

Electrostriction

Constrained Surface: For a constrained surface, the solution to (38) is given by (52) where

$$u_{1_{ST}}(x_1,t) = \frac{\mathrm{Re}\,[\eta]\varepsilon_0(\varepsilon_r-1)(\varepsilon_r+2)I_0}{6\gamma v^2\rho} \cdot [e^{-2\gamma x_1}U(t-T) - e^{-2\gamma x_1}U(t)] \quad (58)$$

and

$$u_{1_{TR}}(x_1,t) = \frac{\mathrm{Re}\,[\eta]\varepsilon_0(\varepsilon_r-1)(\varepsilon_r+2)I_0}{6\gamma v^2\rho} \cdot \Big\{[e^{-2\gamma x_1}\cosh 2\gamma vt]U(t) - [e^{-2\gamma x_1}\cosh 2\gamma v(t-T)]U(t-T) + \left[1-\cosh 2\gamma v\left(t-\frac{x_1}{v}\right)\right]U\left(t-\frac{x_1}{v}\right) + \left[\cosh 2\gamma v\left(t-T-\frac{x_1}{v}\right)-1\right] \cdot U\left(t-T-\frac{x_1}{v}\right)\Big\}. \quad (59)$$

Using (45) to relate stress to displacement, an expression can be obtained for $p(x_1,t)$. A general expression for the pressure wave with the electrostriction forcing function is

$$p(x_1,t) = \frac{\mathrm{Re}\,[\eta]\varepsilon_0(\varepsilon_r-1)(\varepsilon_r+2)I_0}{3} \cdot \Big\{[e^{-2\gamma x_1}\cosh 2\gamma vt]U(t) - [e^{-2\gamma x_1}\cosh 2\gamma v(t-T)]U(t-T) - \left[\sinh 2\gamma v\left(t-\frac{x_1}{v}\right)\right]U\left(t-\frac{x_1}{v}\right) + \left[\sinh 2\gamma v\left(t-T-\frac{x_1}{v}\right)\right]U\left(t-T-\frac{x_1}{v}\right)\Big\}. \quad (60)$$

Free Surface: For a free surface, the solution to (38) is given by (52), where $u_{1_{ST}}(x_1,t)$ is given by (58) and

$$u_{1_{TR}}(x_1,t) = \frac{\mathrm{Re}\,[\eta]\varepsilon_0(\varepsilon_r-1)(\varepsilon_r+2)I_0}{6\gamma v^2\rho} \cdot \Big\{[e^{-2\gamma x_1}\cosh 2\gamma vt]U(t) - \left[\sinh 2\gamma v\left(t-\frac{x_1}{v}\right)\right]U\left(t-\frac{x_1}{v}\right) - [e^{-2\gamma x_1}\cosh 2\gamma v(t-T)]U(t-T) + \left[\sinh 2\gamma v\left(t-T-\frac{x_1}{v}\right)\right]U\left(t-T-\frac{x_1}{v}\right)\Big\} + \frac{2I_r(0,t)}{\rho vc}\Big\{\left[t-\frac{x_1}{v}\right]U\left(t-\frac{x_1}{v}\right) - \left[t-T-\frac{x_1}{v}\right]U\left(t-T-\frac{x_1}{v}\right)\Big\}. \quad (61)$$

Using (46) to relate stress and displacement, the corresponding pressure function is

$$p(x_1,t) = \frac{\mathrm{Re}\,[\eta]\varepsilon_0(\varepsilon_r-1)(\varepsilon_r+2)I_0}{3} \cdot \Big\{[e^{-2\gamma x_1}\cosh 2\gamma vt]U(t) - \left[\cosh 2\gamma v\left(t-\frac{x_1}{v}\right)\right]U\left(t-\frac{x_1}{v}\right) - [e^{-2\gamma x_1}\cosh 2\gamma v(t-T)]U(t-T) + \left[\cosh 2\gamma v\left(t-T-\frac{x_1}{v}\right)\right]U\left(t-T-\frac{x_1}{v}\right)\Big\} + \frac{2I_r(0,t)}{c}\left[U\left(t-\frac{x_1}{v}\right) - U\left(t-T-\frac{x_1}{v}\right)\right]. \quad (62)$$

IV. Fourier Transforms

The spectral content of the displacement and stress functions is particularly pertinent to the study of the microwave hearing phenomenon. Expressions of the magnitudes of the Fourier transforms of the respective functions are given below. Approximations are given when the exact form of the expression is very lengthy. All approximations hold for $2\gamma x_1 \gg 1$.

Constrained Surface

Thermal Expansion:

$$|F[u_1(x_1,t)]| = \frac{12\sqrt{2}\,\alpha v^2\gamma^2 I_0}{\rho S} \frac{[1-\cos(\omega T)]^{1/2}\left(1+e^{-2\gamma x_1}\left(e^{-2\gamma x_1}-2\cos\left(\omega\frac{x_1}{v}\right)\right)\right)^{1/2}}{\omega^4+4\omega^2 v^2\gamma^2} \quad (63)$$

$$|F[p(x_1,t)]| \simeq \frac{12\sqrt{2}\,\alpha v^3\gamma^2 I_0}{S}\left[\frac{(1-\cos(\omega T))^{1/2}}{\omega^3+4\omega v^2\gamma^2}\right]. \quad (64)$$

Electrostriction:

$$|F[u_1(x_1,t)]| = \frac{2\sqrt{2}\,\gamma\ \mathrm{Re}\,[\eta]\varepsilon_0(\varepsilon_r-1)(\varepsilon_r+2)I_0}{3\rho} \cdot\left[\frac{[1+\cos(\omega t)]^{1/2}\left[1+e^{-2\gamma x_1}\left(e^{-2\gamma x_1}-2\cos\left(\frac{\omega x_1}{v}\right)\right)\right]^{1/2}}{\omega(\omega^2+4v^2\gamma^2)}\right] \quad (65)$$

$$|F[p(x_1,t)]| \simeq \frac{2\sqrt{2}\,\gamma v\ \mathrm{Re}\,[\eta]\varepsilon_0(\varepsilon_r-1)(\varepsilon_r+2)I_0}{3}\left[\frac{[1+\cos(\omega T)]^{1/2}}{\omega^2+4v^2\gamma^2}\right]. \quad (66)$$

Free Surface:

As noted above, when the interface surface is considered to be free, an additional forcing function on the material appears in the form of the surface-radiation-pressure boundary condition.

The surface radiation pressure will be treated as a separate forcing function in the Fourier transforms below. For the *thermal-expansion* forcing function,

$$|F[u_1(x_1,t)]| = \frac{6\sqrt{2}\,\alpha v\gamma I_0}{\rho S}\left[\frac{[1-\cos(\omega t)]^{1/2}}{\omega(\omega^2+4v^2\gamma^2)}\right] \quad (67)$$

$$|F[p(x_1,t)]| \simeq \frac{6\sqrt{2}\,\alpha v^2\gamma I_0}{S}\left[\frac{[1-\cos(\omega T)]^{1/2}}{\omega^2+4v^2\gamma^2}\right]. \quad (68)$$

For the *electrostriction* forcing function,

$$|F[u_1(x_1,t)]| \simeq \frac{4\sqrt{2}\,\gamma^2 v\ \mathrm{Re}\,[\eta]\varepsilon_0(\varepsilon_r-1)(\varepsilon_r+2)I_0}{3} \cdot\left[\frac{[1-\cos(\omega t)]^{1/2}}{\omega^2(\omega^2+4v^2\gamma^2)}\right] \quad (69)$$

$$|F[p(x_1,t)]| \simeq \frac{4\sqrt{2}\,\rho\gamma^2 v^2\ \mathrm{Re}\,[\eta]\varepsilon_0(\varepsilon_r-1)(\varepsilon_r+2)I_0}{3} \cdot\left[\frac{[1-\cos(\omega T)]^{1/2}}{\omega(\omega^2+4v^2\gamma^2)}\right]. \quad (70)$$

For the *surface-radiation-pressure* boundary condition,

$$|F[u_1(x_1,t)]| = \frac{2\sqrt{2}\,I_r(0,t)}{\rho vc}\left[\frac{[1-\cos(\omega T)]^{1/2}}{\omega^2}\right] \quad (71)$$

$$|F[p(x_1,t)]| = \frac{2\sqrt{2}\,I_r(0,t)}{c}\left[\frac{[1-\cos(\omega T)]^{1/2}}{\omega}\right]. \quad (72)$$

V. An Example

From the above equations, it is possible to compute both the displacement and stress waveforms in a semi-infinite fluid irradiated with a square microwave pulse. As an example, consider the result of irradiating physiological saline with a 10-μs-wide 3.0-GHz microwave pulse with 27 W/cm^2 incident power density. For physiological saline, the surface transmitted power density would then be about 10 W/cm^2. The appropriate physical constants for this specific case are listed in Table I.

At a distance of 10 cm inside the fluid, the peak displacement and pressure values were computed for the two bulk mechanisms in the constrained-surface case and the two bulk mechanisms plus the surface radiation pressure in the free-surface case. These peak values are listed in Table II.

Some representative waveforms are shown in Figs. 1–4. Figs. 1 and 2 show, respectively, the pressure versus time waveforms due to thermal expansion alone for the constrained- and free-surface boundary conditions. Figs. 3 and 4 show the respective magnitude plots of the Fourier transforms of Figs. 1 and 2.

VI. Discussion

Figs. 1 and 2 are in general agreement with the experimental results previously obtained by Foster and Finch [11], Oswald *et al.* [10], Carome *et al.* [2], Gournay [4], Bushnell and McCloskey [7], and White [9]. References [2], [4], and [9], have presented experimental results of stress development in liquids by irradiation with lasers. Reference [7] has presented similar experimental results using solids irradiated with lasers, while [10] has presented experimental results of displacement in solids due to electron-beam irradiation. Finally, both [9] and [11] have obtained experimental results for microwave induced elastic waves in liquids. The results of Foster and Finch [11], in particular, tend to support the assumptions made in arriving at (36) above.

TABLE I
CONSTANTS USED FOR COMPUTATION OF DISPLACEMENT AND STRESS WAVEFORMS IN PHYSIOLOGICAL SALINE

SYMBOL	PROPERTY	UNITS	VALUE
α	coeff. of linear thermal expansion	$(^{o}C)^{-1}$	10^{-4}
ρ	volume density	gm/cm^3	1.0
v	speed of sound	cm/sec	$1.5(10^5)$
S	specific heat	$ergs/gm^{o}C$	$4.19(10^7)$
γ_{3GHz}	reciprocal of the skin depth	cm^{-1}	0.62
c	speed of light	cm/sec	$3(10^{10})$
ϵ_r	relative dielectric constant of physiological saline	dimensionless	80
η	intrinsic impedance of physiological saline	ohms	44.1 + j.77

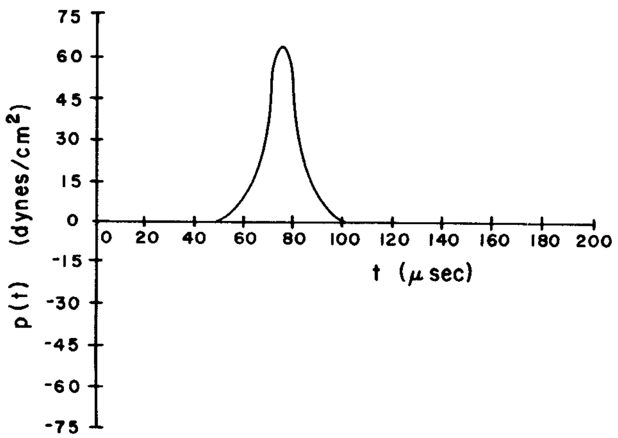

Fig. 1. Pressure versus time for the thermal-expansion mechanism—constrained-surface case.

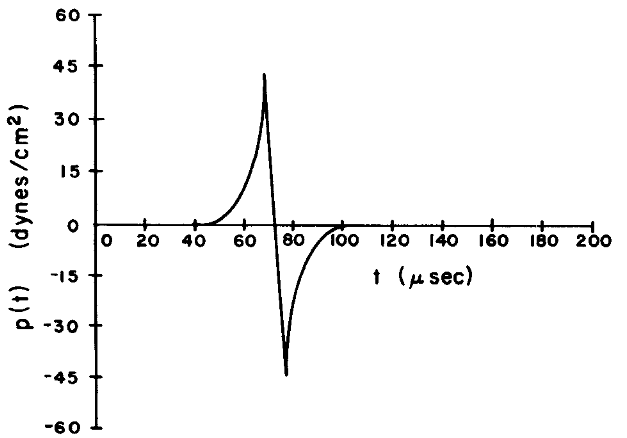

Fig. 2. Pressure versus time for thermal-expansion mechanism—free-surface case.

Figs. 3 and 4 illustrate the low-pass characteristics of the electromagnetic-to-elastic wave conversion process via the thermal-expansion mechanism. Although not treated in the above analysis, consideration of the losses due to

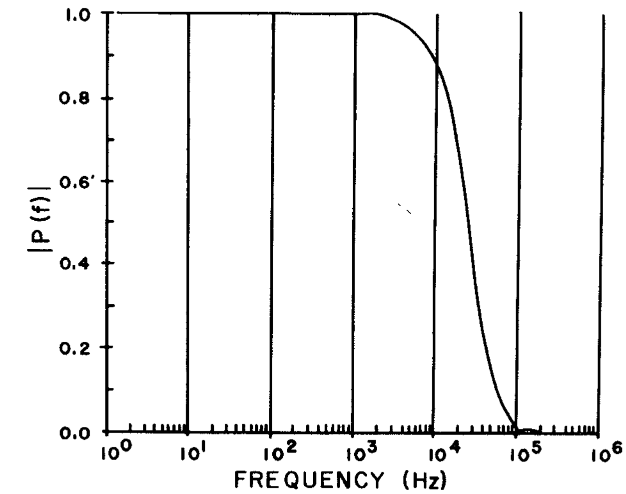

Fig. 3. Normalized magnitude plot of the Fourier transform of pressure for the thermal-expansion mechanism—constrained-surface case.

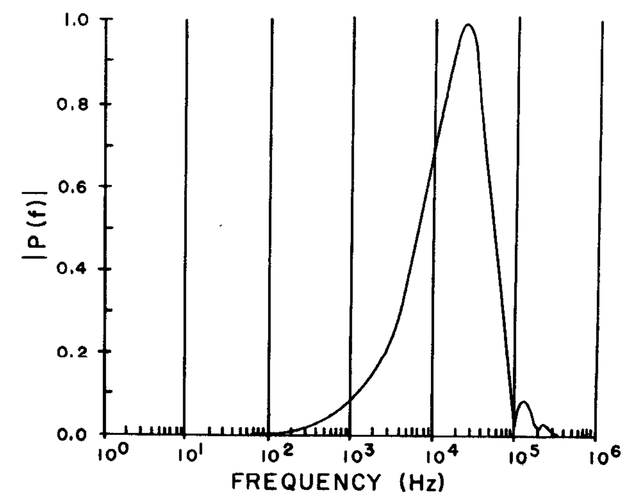

Fig. 4. Normalized magnitude plot of the Fourier transform of pressure for the thermal-expansion mechanism—free-surface case.

TABLE II
PEAK VALUES DISPLACEMENT AND PRESSURE AT $x = 10$ cm INSIDE PHYSIOLOGICAL SALINE IRRADIATED WITH A 3-GHz 10-μs-WIDE PULSE. $I_0 = 10$ W/cm^2 (PEAK)

Forcing Function	Form of Solution	Peak Value	Units
A. Constrained Surface Boundary Condition			
Thermal Expansion	Displacement	$7.17(10^{-9})$	cm
Electrostriction	Displacement	$1.8(10^{-11})$	cm
Thermal Expansion	Pressure	65.04	dynes/cm^2
Electrostriction	Pressure	$3.55(10^{-1})$	dynes/cm^2
B. Free Surface Boundary Condition			
Thermal Expansion	Displacement	$5.13(10^{-9})$	cm
Electrostriction	Displacement	$4.17(10^{-11})$	cm
Surface Radiation Pressure	Displacement	$7.41(10^{-13})$	cm
Thermal Expansion	Pressure	44.22	dynes/cm^2
Electrostriction	Pressure	$5.09(10^{-1})$	dynes/cm^2
Surface Radiation Pressure	Pressure	$1.111(10^{-2})$	dynes/cm^2

dispersive elastic wave attenuation would tend to further attenuate the high-frequency components of the displacement and pressure waves.

Although the theoretical analysis of the thermal-expansion mechanism presented in this paper is in general agreement with analyses presented elsewhere, some differences should be pointed out. Our analysis of the thermal-expansion mechanism for the one-dimensional problem results in a nonhomogeneous wave equation (equation (19)) with a forcing function greater by a factor of three than that found in previous analyses [2], [4], [9]. Essentially our analysis shows that the volume coefficient of thermal expansion (3α) should be employed in the one-dimensional case instead of the linear coefficient of thermal expansion (α). With the exception of this constant, our analysis results in essentially the same traveling wave solutions as presented by Carome *et al.* [2] and Gourney [4]. Our solutions are more complete, however, in that both the stationary and traveling components are considered. Our solutions are also presented in a form more readily adapted to computing actual waveforms in that no changes in the variables x_1 and t are necessary to interpret results.

A general theoretical analysis of stress wave generation by electrostriction is also presented with complete solutions for the one-dimensional special case. In addition, complete solutions are given for the one-dimensional case where surface forces due to radiation pressure are present. Fourier transforms of all traveling wave solutions are presented as an aid in computing the spectral content of the electromagnetically induced acoustic signals. The spectra of induced acoustic waveforms is of particular usefulness in the study of the microwave hearing phenomenon.

Applications of the phenomena discussed in this paper have been described elsewhere [1]–[11], but particularly by White [8], [9]. These phenomena can be used to measure properties of the input electromagnetic radiation, properties of the irradiated material, and as a transducer for generation of acoustic signals in elastic materials.

VII. CONCLUSIONS

Three different physical transduction mechanisms for converting electromagnetic energy to acoustic energy have been studied in the above analysis. A numerical example was considered in which physiological saline was irradiated with pulsed microwave energy and the resulting displacements, pressures, and magnitude plots of the Fourier transform of these quantities were computed for each of the three transduction mechanisms. For the example of an irradiated semi-infinite layer of physiological saline, the computed peak pressure amplitude due to thermal expansion is much greater than the peak pressure generated by either electrostriction or surface radiation pressure.

For the constrained-surface case, a 10-μs-wide 3.0-GHz pulse having an incident intensity of about 27 W/cm^2 resulted in a computed peak pressure of approximately 65 dyn/cm^2 (see Fig. 1). Moreover, the spectral content of the waveform is predominately below 20 kHz as shown in Fig. 3. Such an elastic wave, if generated in a human head, might reach the cochlea via bone conduction causing a distinct auditory sensation.

However, the human head is not a semi-infinite layer of physiological saline, but rather an aspherical multilayer body with a semirigid surface. The acoustic properties of the various tissue layers of the head are not well known at audio frequencies. The above analysis considered an elastic material in which only longitudinal acoustic waves are generated and propagated. For analysis of acoustic pulses generated in the human head, it will be necessary to also consider shear wave generation and propagation. The characteristics of the acoustic signal which reaches the ear are also likely to be determined in part by the natural frequencies (acoustic resonances) of the head itself. Until these aspects are considered in detail, the above work provides a useful comparison of three different electromagnetic-to-acoustic energy transduction processes.

Appendix

In the above analysis it was stated that under certain conditions (33) reduced to (34), resulting in a simplified expression for the excess temperature function. In what follows this simplification is justified.

Equation (33) may be rewritten as follows:

$$\begin{aligned}\theta(x,t) = {} & \frac{2I_0}{K}\phi\left[-\left(\frac{x}{2\phi}\right)\left(1-\operatorname{erf}\left(\frac{x}{2\phi}\right)\right)+\frac{1}{\sqrt{\pi}}e^{-(x/2\phi)^2}\right] \\ & -\frac{I_0}{2\gamma K}e^{-2\gamma x} \\ & +\frac{I_0}{4K\gamma}e^{(4\gamma^2\phi^2-2\gamma x)}\left[1-\operatorname{erf}\left(2\gamma\phi-\frac{x}{2\phi}\right)\right] \\ & +\frac{I_0}{4K\gamma}e^{(4\gamma^2\phi^2+2\gamma x)}\left[1-\operatorname{erf}\left(2\gamma\phi+\frac{x}{2\phi}\right)\right] \qquad \text{(A1)}\end{aligned}$$

where the identities $\operatorname{ierfc}(z) = -z\operatorname{erfc}(z) + (1/\sqrt{\pi})e^{-z^2}$ and $\operatorname{erfc}(z) = 1 - \operatorname{erf}(z)$ have been used. As stated above, the two conditions necessary for (33) to simplify to (34) are

$$\frac{x}{2\phi} \gg 1 \qquad \text{(A2)}$$

and

$$4\gamma^2\phi^2 \ll 1. \qquad \text{(A3)}$$

From these two conditions, it follows that

$$\frac{x}{2\phi} \gg 2\gamma\phi. \qquad \text{(A4)}$$

Applying (A4) to (A1) results in

$$\begin{aligned}\theta(x,t)\Big|_{x/2\phi \gg 2\gamma\phi} \simeq {} & \frac{2I_0}{K}\phi\left[-\left(\frac{x}{2\phi}\right)\left(1-\operatorname{erf}\left(\frac{x}{2\phi}\right)\right)\right. \\ & \left.+\frac{1}{\sqrt{\pi}}e^{-(x/2\phi)^2}\right]-\frac{I_0}{2\gamma K}e^{-2\gamma x}+\frac{I_0}{4K\gamma} \\ & \cdot\left\{e^{(4\gamma^2\phi^2-2\gamma x)}\left[1+\operatorname{erf}\left(\frac{x}{2\phi}\right)\right]\right. \\ & \left.+e^{(4\gamma^2\phi^2+2\gamma x)}\left[1-\operatorname{erf}\left(\frac{x}{2\phi}\right)\right]\right\} \qquad \text{(A5)}\end{aligned}$$

where the identity $\operatorname{erf}(-z) = -\operatorname{erf}(z)$ has been used. For large values of its argument z,

$$\lim_{z\to\infty}\operatorname{erf}(z) = 1.$$

For example, for $z \geq 10$, $1 > \operatorname{erf}(z) > 1 - 2\times 10^{-44}$ [26]. Hence, applying condition (A2) to (A5)

$$\theta(x,t)\Big|_{\substack{x/2\phi \gg 2\gamma\phi \\ \text{and } x/2\phi \gg 1}} \simeq \frac{I_0}{2\gamma K}e^{-2\gamma x}[e^{4\gamma^2\phi^2}-1]. \qquad \text{(A6)}$$

Expanding the term in brackets into a Taylor series and applying condition (A3)

$$\begin{aligned}\theta(x,t)\Big|_{\substack{x/2\phi \gg 1 \\ \text{and } 4\gamma^2\phi^2 \ll 1}} & \simeq \frac{I_0}{2\gamma K}e^{-2\gamma x}[1+(4\gamma^2\phi^2)-1] \\ & = \frac{2\gamma I_0\phi^2}{K}e^{-2\gamma x} \qquad \text{(A7)}\end{aligned}$$

where the higher order terms in the series expansion have been dropped because of condition (A3).

Finally, noting that

$$\phi = (\kappa t)^{1/2} = \left(\frac{K}{\rho S}t\right)^{1/2} \qquad \text{(A8)}$$

(A7) reduces to

$$\theta(x,t)\Big|_{\substack{x/2\phi \gg 1 \\ \text{and } 4\gamma^2\phi^2 \ll 1}} \simeq \frac{2\gamma t I_0}{\rho S}e^{-2\gamma x} \qquad \text{(A9)}$$

which is (34).

A question remains as to whether conditions (A2) and (A3), in fact, hold for the case of a liquid irradiated with microwave pulses. For the example considered above,

$$\frac{x}{2\phi} = 13.2268\frac{x}{\sqrt{t}}$$

and $4\gamma^2\phi^2 = 2.1972\times 10^{-3}t$, where x is in centimeters and t is in seconds.

Note that, for this example, condition (A2) holds for $(x/\sqrt{t}) > 1$ and condition (A3) holds for $t < 50$ s. Consequently, for the case treated above, the assumption that (33) reduces to (34) is valid.

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Input Impedance of Coaxial Line to Circular Waveguide Feed

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Abstract—**The expressions for the real and imaginary parts of the input impedance seen by a coaxial line driving a thin cylindrical probe in a dominant TE_{11} mode circular waveguide are derived. The analysis is carried out by assuming that the cylindrical post is replaced by a curvilinear strip having maximum width equal to the diameter of the probe. Theoretical results on input VSWR and input impedance seen by a coaxial line are in close agreement with experimental data.**

I. Introduction

Electromagnetic energy is coupled to a waveguide by means of a probe or loop radiators driven from a source through a coaxial line. A rigorous solution of the problem of a cylindrical probe parallel to the electric field in a rectangular waveguide has been presented by Collin [1]. Harrington [2] has found a method for determining an equivalent network of the junction between a coaxial line and a rectangular waveguide and has determined the resistive part of the input impedance seen by a coaxial line from the stationary formula for the impedance. An analysis for the determination of variation of both resistive and reactive parts of the input impedance for a cylindrical probe exciting a circular cylindrical waveguide has not been reported in the literature.

In this paper, expressions for the real and imaginary parts of the input impedance seen by a coaxial line driving a cylindrical probe in a dominant TE_{11} mode circular waveguide are derived. Assumption of a purely filamentary radial probe leads to a divergent series for the imaginary part of the input impedance [2]. For the purpose of analysis the thin cylindrical probe is assumed to be replaced by a curvilinear metallic strip in the cross section of the waveguide. This assumption simplifies the analysis considerably and leads to an expression for the imaginary part of the input impedance in the form of an infinite series which is convergent. A formula for the input impedance seen by a coaxial line is derived for a circular cylindrical waveguide terminated in a matched load on one side and in a short circuit at a distance L_1 from the probe on the other side. The expressions for the parameters of the equivalent network of the junction are also derived.

The variation of the input impedance with frequency seen by a coaxial line is computed for probe length l, $0.6 \le l \le 0.8$ cm, probe diameter $d = 0.2$ cm, and $0.7 \le L_1 \le 1.0$ cm. The theoretical results on variation of the input impedance seen by a coaxial line and the VSWR in a coaxial line are in close agreement with the experimental data for a radial probe having a diameter equal to the maximum width of the curvilinear strip.

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